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On Inferences in Acceptance Sampling for Generalized Rayleigh Distributed Life Time

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Abstract

For reliability inspection, an effective acceptance sampling plan is always required which can satisfy both the producer's and the consumer's quality and risk. This paper constructs the effective single sampling plan for reliability inspection when the distribution of the failure items follows generalized Rayleigh distributed lifetime. For this purpose, we consider the ratio of index average lifetime and testing time for two values of average lifetime-acceptable and non-acceptable ones and known shape parameter of the distribution. A relationship between index and reliability function is also derived and its use is illustrated by an example.

Keywords

Acceptance Sampling; Quality and Risk; Generalized Rayleigh Distribution; Testing Time; Average Lifetime and Reliability

Introduction

Quality concept revolves around the meeting or exceeding customer expectation applies to the product and service. Managing quality is always crucial and achieving high quality is an ever changing or continuous process; therefore, quality management emphasizes the ideas of working constantly towards improving the quality. It involves every aspects of the company: processes, environment and their people. Quality product always helps to maintain customer satisfaction and loyalty and reduces the risk and cost of replacing faulty goods. Companies can build a reputation for quality by gaining accreditation with a recognized quality standard. That’s why the developing and competitive world now has given great importance to statistical quality control techniques for ensuring the trustworthiness of an item with regards to its lifetime by doing inspections. In this situation, it is necessary to ascertain the visible operating characteristic values of the proposed plan.

Acceptance sampling plan ensures that the lifetime of the product is according to the specified/desired level/standard of the consumer. It is often used to determine the disposition of incoming raw material or parts when 100 percent inspection is destructive, time consuming, or expensive. An effective acceptance sampling plan will allow to set and maintain an acceptable quality standard. In acceptance sampling, inspection is performed on a sample taken from a lot of incoming materials. A decision is made concerning the disposition of the lot based on the information obtained from this sample. In general, three approached could be considered when receiving a lot: (1) no inspection, (2) 100% inspection, and (3) sampling inspection. If the life test of the items follows that the average life of the product is above the specified/desired level/standard, the submitted lot is accepted otherwise the same is rejected.

The item failure is due to natural causes or due to some spurious ones i.e. environment, lack of maintenance actions, mishandling, intensive operational tasks etc. If the item failure occurs after a certain specified period of time, then it will be satisfactory operations. But we cannot explain the exact time of a specified object when it will fail. We are in position to explain it in terms of probabilities and average expected time as its main parameters. Now, the failure behaviour of that specific object is to be modeled and hence chosen as the most suitable class of life distributions describing this time-to-failure phenomenon. The document MILSTD-781 Reliability test: exponential distribution used the ratio $[E(T)]/T_0$, where E(T) is the average lifetime or durability of underlying objects and T_0 is the testing time in the exponential case (Cătuneanu, and Mihalache, 1989; Voda, and Isaic-Maniu, 1994). Later, some inferences for Weibull distribution case have been discussed (Isaic-Maniu and Vodă, 2009). Here,

we are considering the generalized Rayleigh distribution which now becomes an important lifetime distribution in survival analysis and many distributions are its special case. The present study deals with some new results on the index average lifetime/testing time in the construction of acceptance sampling plans for reliability inspection, when time-to-failure distribution is following generalized Rayleigh distribution.

The Model: Generalized Rayleigh Distribution

The Rayleigh distribution is useful in life testing and reliability modeling when age varies with time and its failure rate is a linear function of time. The Rayleigh distribution was first derived for the problems in the field of acoustics (Rayleigh, 1880). It was also applied in life testing of electro-vacuum devices and in communication engineering (Polovko, 1968). Later, an acceptance sampling plan assuming for a generalized Rayleigh distribution has been developed (Dyer and Whisenand, 1973; Tsai and Wu, 2006). Rayleigh distribution has a linearly increasing failure rate which makes it a suitable model for the lifetime of components. A generalized version of the Rayleigh distribution called the generalized Rayleigh distribution (GRD) is derived (Voda, V. Gh. 1976). The *pdf* of this GRD is given by

$$f(t) = \frac{2}{\Gamma(k+1)} \frac{1}{\lambda^{k+1}} t^{2k+1} e^{-t^2/\lambda}, t > 0,\ \theta > 0,\ k \geq 0.$$

where $\Gamma(\cdot)$ is incomplete gamma function as

$$\Gamma(z, a) = \frac{1}{\Gamma(a)} \int_0^z z^{a-1} e^{-z}\ dz$$

This GRD family also includes several important probability distributions as special case.

For example, if $k = 0$ and $\lambda = \theta^2$, it reduces to the one-parameter Rayleigh distribution with density function

$$f(x;\ \theta) = \frac{2}{\theta^2} t \exp\left(-\frac{t^2}{\theta^2}\right);\ t > 0,\ \theta > 0.$$

For, if $k = 1/2$ and $\lambda = \theta^2$, it reduces to the one-parameter Maxwell distribution with density function

$$f(x;\ \theta) = \frac{2}{\theta^3 (2\pi)^{1/2}} t^2 \exp\left(-\frac{t^2}{2\theta^2}\right);\ t > 0,\ \theta > 0.$$

For, if $k = [(a/2) - 1]$ and $\lambda = \tau^2$, it becomes chi-square distribution with 'a' degrees of freedom whose density function

$$f(x; \tau, a) = \frac{1}{2^{(a/2)-1} \tau^a\ \Gamma(a/2)^{1/2}} t^{a-1} \exp\left(-\frac{t^2}{2\tau^2}\right); \quad t > 0, \tau > 0.$$

where '$a \in N$', 'N' the sets of natural numbers.

Background and Assumptions for Reliability Inspection

The characteristic of interest is reliability or durability of underlying items under batch inspection. In order to construct suitable acceptance sampling plans under economical condition, we must take into account of their failure behaviour which is concerned with time. Under the batch inspection, the attributive method never matters the nature of the investigated quality characteristics. But, in case of reliability or durability; the attributive approach does not take care of the following elements: i) assumption about failure time distribution; ii) inspection: complete or censored ones; iii) sampling: with or without replacement; iv) conditions: accelerated or normal testing;

v) the relationship: between testing time (T_0) and the actual operating life of the items; vi) items: repairable or non-repairable;

The nature of attributive method lies in the fact that products are classified into two categories: conforming and nonconforming (defective) ones for some specified criteria. In the case of reliability/durability inspection, this attributive approach ignores the nature of failure behaviour of inspected objects and this could lead to a larger sample to be test. If the items are quite expensive and the specific test in this case is destructive, the procedure appears to be non-economic. If they are non- restoring, then $E(T)$ is just the mean durability and $\bar{T}$, the sample mean is computed with $(t_i)_{1\le i\le n}$, the first and last failure values of the i^{th} item on the test; it is worthless to describe about Mean Time Between Failures. In this case, a method based on average operating time or on hazard rate associated with the failure time model for each attributive instance is useful.

The Ratio $E(T)/T_0$ for Generalized Rayleigh Distribution

Let T be the distribution function of Generalized Rayleigh distribution

$$f(t) = \frac{2}{\Gamma(k+1)}\frac{1}{\lambda^{k+1}}t^{2k+1}e^{-t^2/\lambda}, t>0,\ \theta>0,\ k\ge 0.$$

and cumulative density function is

$$F(t) = 1-\sum_{j=0}^{k}\frac{(t^2/\lambda)^j e^{-t^2/\lambda}}{j!}.$$

where k, a positive integer is called the shape parameter and λ is scale parameter.

The corresponding reliability unction is

$$R(t) = \sum_{j=0}^{k}\frac{(t^2/\lambda)^j e^{-t^2/\lambda}}{j!}$$

and mean value is $E(T) = m\lambda^{1/2}$.

where $m = \Gamma(k+3/2)/\Gamma(k+1)$

Hence, we have

$$\lambda = \left[\frac{E(T)}{m}\right]^2$$

and consequently, we get

$$R(t) = \sum_{j=0}^{k}\left[\left(\frac{mt}{E(t)}\right)^2\left(\frac{mt}{E(t)}\right)^{2j}\exp\left(-\left(\frac{mt}{E(t)}\right)^2\right)\Big/ j!\right]$$

Therefore, for generalized Rayleigh distribution, the ratio $E(T)/t$ depends on its reliability function. If we fix $t=T_0$, we have either to estimate $R(T_0)$ or to fix lower acceptable bound for it. Now

$$R(T_0) = \sum_{j=0}^{k}\left[\left(\frac{mT_0}{E(T_0)}\right)^2\left(\frac{mT_0}{E(T_0)}\right)^{2j}\exp\left(-\left(\frac{mT_0}{E(T_0)}\right)^2\right)\Big/ j!\right]$$

Construction of the Proposed Sampling Plan

The sampling plan is the system of objects $\{(n,c)\,|\,T_0\}$ where n and c are respectively the sample sizes and

acceptance number which is to be determined and T_0 is the previously fixed testing time. The procedure for the construction of a sampling plan has the following assumptions:

a) The items subjected to inspection are non-reparable and its failure time follows GRD;

b) Its size is to be determined with only one sample- no replacement;

c) For a given risk α , there is fixed an acceptable average lifetime $[E(T)]_1$;

d) For a given risk β, there is fixed an non-acceptable average lifetime $[E(T)]_2$;

e) The testing time T_0 smaller than the actual operating life of the underling items is fixed.

Then decision on the lot is taken as follows: submit to the specific reliability/durability test of a sample size n drawn randomly from a lot of size N $(n < N)$, during a period of units of T_0; record then the number (d) of failed elements in the interval $[0, T_0]$; if $d \le c$, then the lot is accepted - otherwise, if $d > c$, then the lot is rejected. The values of n and c are determined via the OC - function (Operative Characteristic) of the plan which has the function

$$L(p) = \sum_{d=0}^{c} \frac{1}{d!}(np)^d \; e^{-np}$$

where $d! = 0,1,2,\ldots,c$. and p is the defective fraction in the lot given by $p = 1 - \sum_{j=0}^{k} \frac{(t^2/\lambda)^j e^{-t^2/\lambda}}{j!}$, d is the number of failed elements during the testing period T_0 (Grant, and Leavenworth, 1988).

Let's define two values for p (say, p_1 and p_2) for which $L(p_1) = 1-\alpha$ and $L(p_2) = 1-\beta$. Using the ratios $[E(T)]_1 / T_0$ and $[E(T)]_2 / T_0$, we obtain a system which provides the values of n and c of the specified plan. Table 1 presents some values for n and c for the generalized Rayleigh lifetime. The input data can be the following quantities: $100T_0 / [E(T)]_1$ for which $L(p_1) = 0.95$ and $100T_0 / [E(T)]_2$ $L(p_2) = 0.10$ (the first figure is given in brackets). This approach avoids the knowledge of $R(T_0)$ since the input values are only T_0 and $[E(T)]_{1,2}$ which are fixed previously taking into account of the pre-specific case at hand.

TABLE 1: VALUES OF SINGLE SAMPLING PLAN $[(n, c)|T_0]$ FOR k=1 AND THE INPUT RATIOS $100T_0 / [E(T)]_{1,2}$

c	n Values of $100T_0 / [E(T)]_1$ for which $L(p_2) = 0.10$			
	100	**50**	**25**	**15**
0	3 (11)	11 (6.3)	40 (2.8)	113 (1.9)
1	6 (25)	16 (10)	71 (7)	214 (3.8)
2	8 (33)	27 (15)	101 (7.2)	276 (5.2)
3	9 (35)	31 (18)	119 (9.1)	343 (6.1)

An Illustrative Example

Assume that we have an acceptable durability $[E(T)]_1 = 4000$ hours and a non-acceptable one as $[E(T)]_2 = 800$ hours. Testing time was fixed at the value $T_0 = 200$ hours. The usual consumer risk $\alpha = 0.05$ and producer's risk $\beta = 0.10$

Therefore, to find the plan, we have

$$\frac{100\,T_0}{[E\,(T)]_2} = \frac{100.\times 200}{800} = 25 \text{ and } \frac{100\,T_0}{[E\,(T)]_1} = \frac{100.\times 200}{4000} = 5$$

From the table, the nearest value of $100T_0/[E(T)]_1$ for $100T_0/[E(T)]_2 = 25$ is 7 and hence for the couple 50 (7). We choose $n = 71$ sampling units with the acceptance number $c = 1$. The sampling plan is then {(71, 1) | 200} and as a consequence, we shall test n = 71 items on a period of 200 hours and record d - the number of failed items. If $d = 0$ or 1, we shall accept the lot - otherwise we shall reject it.

Conclusions

Acceptance sampling plans are being widely used in industry to determine whether a specific batch of manufactured or purchased items satisfy a pre-specified quality to its customer. For determining economical acceptance sampling plan, we have to fix the time and then estimate the reliability or we can fix the lower acceptable bound. This method will help us in deciding the sample number for inspection and a small sample number can be selected for different combination of the parameters for the plan.

REFERENCES

[1] Cătuneanu, V., M. and Mihalache, A., N. 1989: Reliability Fundamentals, Elsevier, Amsterdam (Fundamental Studies in Engineering, No.10 - Translated from Romanian by A. N. Mihalache).

[2] Voda, V., Gh., Isaic-Maniu, Al. 1994: The power distribution as a time-to-failure model, Economic Computation and Economic Cybernetics Studies and Research, 28, (1-4), 41 - 51.

[3] Isaic Maniu, Al- Vodă, V. Gh. 2009: Some inferences on ratio average lifetime/testing time in acceptance sampling plans for reliability inspection. Reliability: Theory & Applications, 4(3), 51-57.

[4] Rayleigh, J. W. S. 1880: On the resultant of a large number of vibrations of the same pitch and of arbitrary phase, Philosophical Magazine, 5 th series, 10, 73-78.

[5] Polovko, A. M. 1968: Fundamentals of Reliability Theory: Academic Press, New York.

[6] Dyer, D. D. and Whisenand, C. W. 1973: Best linear unbiased estimator of the parameter of the Rayleigh distribution-Part I: Small sample theory for censored order statistics, IEEE Transaction on Reliability, 22, 27-34.

[7] Tsai, Tzong-Ru and Wu, Shuo-Jye 2006: Acceptance sampling based on truncated life tests for generalized Rayleigh distribution, Journal of Applied Statistics, 33(6), 595-600.

[8] Voda, V. Gh. 1976: Inferential procedures on a generalized Rayleigh variate-I, Aplikace Mathematiky, 21, 395-412.

[9] Grant, E.L. and Leavenworth, R.S. 1988: Statistical Quality Control, 6th Edition Mc. Graw Hill Book, Co., New York.

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